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Hertz contact stress

The contact stress between curved bodies — point and line contact formulas, the effective modulus and radius, the subsurface shear that drives pitting, and allowable limits.

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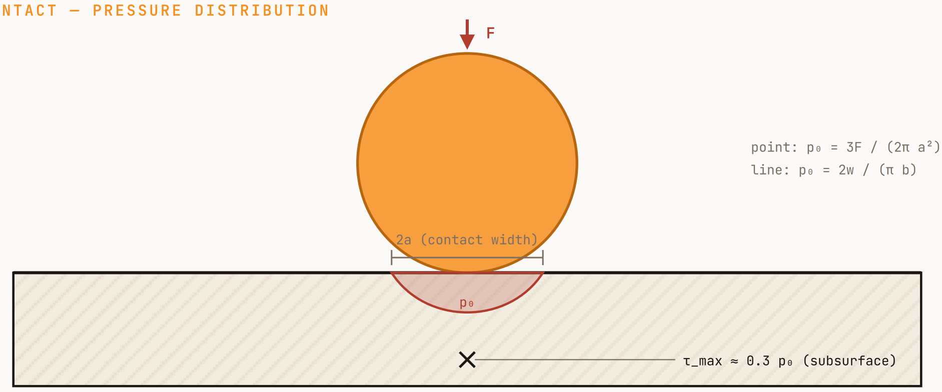
ABSTRACT

When curved bodies touch — a ball on a race, a gear tooth on its mate, a cam on a follower, a wheel on a rail — they contact over a very small area, so the local (Hertzian) stress is far higher than the nominal load suggests. The peak contact pressure and the shear it produces just below the surface are what limit rolling elements.

Section 1 explains Hertz contact. Section 2 is point contact (spheres/balls). Section 3 is line contact (cylinders/rollers). Section 4 is the effective modulus and radius. Section 5 covers subsurface shear and allowable stress. Section 6 is design notes and a checklist.

HERTZIAN CONTACT — PRESSURE DISTRIBUTION



TWO CURVED BODIES TOUCH OVER A TINY PATCH, SO THE LOCAL PRESSURE IS ENORMOUS AND CONCENTRATED. THE PEAK PRESSURE P_0 AND THE SUBSURFACE SHEAR IT CREATES GOVERN PITTING IN BEARINGS, GEARS AND CAMS.

CONTENTS

1. What Hertzian contact is

Two non-conforming curved surfaces pressed together deform elastically into a small contact patch — a circle (point contact) or a narrow strip (line contact). Across that patch the pressure is **semi-elliptical**, peaking at **p_0** in the centre. Because the area is tiny, p_0 is very large, and the maximum shear stress sits a little **below** the surface — which is where rolling-contact fatigue (pitting/spalling) initiates.

p_0	Maximum (centre) contact pressure — the value you check
Point contact	Ball-on-flat / ball-in-race / sphere-on-sphere → circular patch radius a
Line contact	Cylinder-on-flat / roller-in-race → strip of half-width b, load per length w
E*	Effective (reduced) modulus combining both bodies' elasticity
R*	Effective (relative) radius combining both bodies' curvatures

2. Point contact (spheres / balls)

For two spheres (or a ball on a flat, $R_2 \to \infty$):

- **Contact radius** `a = (3 F R* / 4 E*)^{1/3}`
- **Peak pressure** $p_0 = 3F / (2 \pi a^2)$ **(= 1.5 × the average pressure)**

p_0 rises only with the **cube root** of load, so doubling the load raises peak pressure ~26% — but it also rises fast as radius shrinks, which is why small balls and sharp crowns are highly stressed.

3. Line contact (cylinders / rollers)

For two parallel cylinders carrying load per unit length $w = F / L$:

- **Contact half-width** $b = \sqrt{4 w R^* / (\pi E^*)}$
- **Peak pressure** $p_0 = 2 w / (\pi b) = \sqrt{w E^* / (\pi R^*)}$

Line contact spreads load along the roller, so for the same total load it gives a lower p_0 than point contact — the reason roller bearings carry more than ball bearings of the same size. Watch **edge loading**: misalignment concentrates w at the roller ends, so rollers are crowned.

4. Effective modulus and radius

Both contact bodies share the deformation, combined as:

- $1/E^* = (1 - \nu_1^2)/E_1 + (1 - \nu_2^2)/E_2$
- $1/R^* = 1/R_1 \pm 1/R_2$

  use **+** for two convex surfaces, **–** for a convex body in a concave one (a ball in its race), which makes R* large and lowers stress.

So a ball seated in a conforming raceway groove is far less stressed than the same ball on a flat. Steel-on-steel: $E^* \approx 113$ GPa ($E = 207$ GPa, $\nu = 0.3$ each).

5. Subsurface shear and allowable stress

- **The maximum shear is ≈ 0.30 · p_0, located about 0.48 a (point) or 0.78 b (line) below the surface. Rolling fatigue cracks start there and work to the surface as pits.**
- **Static limit (brinelling): permanent denting begins around p_0 ≈ 4000–4200 MPa for hardened bearing steel**
  the basis of the static load rating C_0.
- **Cyclic limit (pitting): far lower; case-hardened gear/bearing flanks are typically designed to ~1300–1700 MPa contact stress for long life (per ISO 6336 / bearing L10).**

SITUATION	INDICATIVE P_0 LIMIT
Hardened bearing steel, static (brinelling)	~4000–4200 MPa
Case-hardened gear flank, long life	~1300–1700 MPa
Through-hardened steel, moderate cycles	~1000–1400 MPa

Values are indicative — use ISO 6336 (gears) or bearing L10 ratings for real life.

6. Design notes and checklist

- Lower p_0 by increasing the effective radius (crowning, conforming grooves), spreading to line contact, or reducing load.
- Raise the limit with harder, cleaner steel (case-hardening, low inclusions) and a good EHL oil film (a full film dramatically extends pitting life).
- Mind subsurface cleanliness
  inclusions at the max-shear depth are crack nuclei; bearing steels are vacuum-degassed for this reason.
- Checklist: find E* and R* → compute a/b and p_0 (point or line) → compare to the static limit (brinelling) and the cyclic limit (pitting) for the material and life → if high, add crowning/conformity, harden, or improve lubrication. Pairs with the
  Bearing
  ,
  Gear
  and
  Fatigue
  references.